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Algorithm for the Iterative Design of Observer Field Tests

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ABSTRACT

In a previous paper, I described a technique for designing an observer test in an iterative manner. In field tests to compare the observability of combat vehicles, the test designer must select the optimum number of observation opportunities in order to balance collecting enough data to draw valid conclusions against the high cost of supporting vehicles and personnel at a test site. The test designer can select the number of observations, N, so that a given experimental difference in detectability will be statistically significant at a given confidence level. Alternatively, the test designer can select N so that the probability of rejecting a given underlying difference in detectability is less than a given amount. The test designer, however, generally lacks key parameters for the efficient design of the test. Namely, the designer lacks the detection probabilities of the vehicles at each range. The standard deviation of the difference in detection probability depends upon the detection probability itself. Therefore, the test designer must select the number of observations for each range based upon the conservative assumption that the probabilities are near 50%, the probability for the maximum standard deviation. In the previous paper, an iterative technique of test design was described. In this technique, the test designer modifies the test matrix as the test progresses. Early test results yield estimates of the probability of detection for each vehicle at each range. Based on these estimates, the test designer reallocates the number of observations among the ranges, improving the efficiency of the test.

In this paper, I present an algorithm to implement this iterative technique. Initially, observation opportunities are equally distributed among the ranges of vehicle sites. But, as the test progresses, the observation opportunities are redistributed among the ranges according to estimates of detection probability. For example, the fraction of opportunities at the range near 50% probability is increased while the fraction near 10% probability is decreased. The algorithm handles the special cases that occur when the estimated probability is 1 or 0.

1. THE EXPERIMENTAL SITUATION

Figure 1 illustrates a typical test setup for a test of observability. Observers are stationed at a fixed site and attempt to detect a vehicle in their field of view. For each observation opportunity, the test personnel record the number of detections. In analyzing the data, the analyst groups the observations into range bins and compares the proportion of detections for each test vehicle.

For such a field test, the test designer must select the optimum number of observation opportunities at each range. The designer must balance collecting enough data to draw valid conclusions against the high cost of supporting vehicles and personnel at a test site.

2. DESIGNING THE EXPERIMENT

The test designer selects the number of observations in order to achieve two objectives. First, the number of observations must be large enough so that the probabilities of errors are below the desired level. And, second, the number of observations must be as small as possible to minimize testing time and cost.

Table I illustrates the types of errors that can occur in drawing conclusions from an observation test. The goal of an observation test is to judge whether the observed difference in detection probability, Pd, is unusual enough to reject the null hypothesis that Vehicle A and Vehicle B have the same Pd. If the analyst concludes that the null hypothesis cannot be rejected, then either the analyst has made a correct decision or a Type II error. On the other hand, a decision to reject the null hypothesis will be either correct or a Type I error. In terms of countermeasure effectiveness, a Type I error is an erroneous conclusion than an ineffective countermeasure is effective, while a Type II error miss judges an effective countermeasure.

In a previous paper [1], I discussed the number of observation opportunities required to control the probability of these errors. For the case of an average Pd of 0.5, Figure 2 plots the number of observations required to maintain the probability of Type I and Type II errors at less than 5%. For example, 85 observations of each vehicle are adequate for an observed 0.15 difference to be significant with less than 5% chance of a Type I error. On the other hand, for less than 5% chance of committing a Type II error when the underlying probabilities differ by 0.15, the requirement is 316 observation opportunities per vehicle.

3. AN ITERATIVE TECHNIQUE FOR TEST DESIGN

An iterative technique can reduce the number of observation opportunities required for a test. Observation data is described by the binary distribution. But the binary distribution has the property that the standard deviation depends upon the probability. If p is the probability and N is the number of trials, then the standard deviation of the number of successes, σ, is given by

$$\sigma_{\text{detections}} = \sqrt{Np(1-p)} \tag{1}$$

And the standard deviation of the proportion of successes is given by

$$\sigma_{\text{proportion}} = \frac{\sigma_{\text{detections}}}{N} = \sqrt{\frac{p(1-p)}{N}} \tag{2}$$

Figure 3 plots Equation 2 for an N of 100, 200 and 300. Figure 4 plots the same equation normalized to its maximum at p = 0.5.

Figure 5 further shows how the number of trials changes with probability if the standard deviation is held fixed. For example, in comparison to a probability of 0.5, only 60% as many trials are need at a probability of 0.1.

To use this property of the binary distribution, the test designer modifies the test matrix as the test progresses. Early test results yield estimates of the probability of detection for each vehicle at each range. Based on these estimates. the test designer reallocates the number of observations among the ranges, improving the efficiency of the test design. Figure 5 illustrates a sample of the improvement in efficiency that this technique can achieve. Initially the test designer would have selected the number of observations at each test range by assuming the worst-case value of Pd = 0.5. But as the test progresses, the designer would use estimates of Pd from the early measures to redesign the test. For example, at range 20, the designer would reduce the number of observations from 314 to 251. Overall in this sample test, the test designer would reduce the number of observations from 1570 to 1193, a reduction of 24%.

4. IMPLEMENTATION OF THE ITERATIVE TECHNIQUE FOR TEST DESIGN

A. Algorithm for Fixed Number of Observation Opportunities

The first step is selection of the number of observation opportunities for each range based on the conservative assumption of a Pd of 50% at each range. From figures x and y, the designer selects No to reduce the probabilities of Type I and II errors to acceptable levels.

Next, the designer calculates estimates of P_d^i, the average probability of detection at range R_i, based on the results of the first day of observations, as follows:

$$P_d^i = \frac{D_A + D_B}{N_A + N_B} \tag{3}$$

where D_A and D_B are the numbers of detections of Vehicles A and B and N_A and N_B are the number of observation opportunities for Vehicles A and B.

To account for special cases when there are no detections or 100% detections, Equation 3 should be modified to prevent later setting the number of observation opportunities to zero. The following form covers these special cases,

$$\begin{gathered} \text{If } (D_A+D_B)/(N_A+N_B) < 0.1, \text{ then } P_d^i = 0.1 \\ \text{If } (D_A+D_B)/(N_A+N_B) > 0.9, \text{ then } P_d^i = 0.9 \\ \text{Otherwise, } P_d^i = \frac{D_A + D_B}{N_A + N_B} \end{gathered} \tag{4}$$

Next, beginning with Equation 3, the designer calculates N_i, the new number of observation opportunities for range R_i:

$$\sigma = \sqrt{\frac{P_d^i(1-P_d^i)}{N_i}} \tag{5}$$

Note that σ is the same for all ranges. Solving for N_i yields,

$$N_i = \frac{P_d^i\left(1 - P_d^i\right)}{\sigma^2} \qquad (6)$$

And, keeping the total number of observation opportunities constant at N_T, then

$$N_T = \sum_{i=1}^{R} N_i = \frac{1}{\sigma^2}\sum_{i=1}^{R} P_d^i\left(1 - P_d^i\right) \qquad (7)$$

Solving for σ^2, gives

$$\sigma^2 = \frac{1}{N_T}\sum_{i=1}^{R} P_d^i\left(1 - P_d^i\right) \qquad (8)$$

And

$$N_i = N_T \frac{P_d^i\left(1 - P_d^i\right)}{\sum_{j=1}^{R} P_d^j\left(1 - P_d^j\right)} \qquad (9)$$

Now, the test continues with the modified number of opportunities for observation at each range. At the end of each succeeding day, the designer can recalculate the N_i's in the same way to refine the design.

B. Algorithm for Minimal Number of Observation Opportunities

As an alternative to keeping the total number of opportunities for observation fixed, the designer can decrease the number of opportunities for P_d's that differ from 50%. With this criterion, the new number of observations can be calculated by using Equation 2 to keep the standard deviations constant,

$$\sqrt{\frac{P_d^i\left(1 - P_d^i\right)}{N_i^{new}}} = \sqrt{\frac{0.5(1 - 0.5)}{N_i^{initial}}} \qquad (10)$$

Solving for N_i^{new} gives,

$$N_i^{new} = \frac{N_i^{initial}}{0.25} P_d^i\left(1 - P_d^i\right) \qquad (11)$$

Again, at the end of each succeeding day, the designer can recalculate the N_i's to refine the design.

5. CONCLUSION

In this paper, I have presented an iterative technique of using early test results in an observation test to improve the overall efficiency of the test design.

6. REFERENCE

1. J. G. Bennett, " Proceedings of the 2001 Ground Target Modeling & Validation Conference, Houghton, MI, August 2001.

Possible Outcomes of Hypothesis Testing

	Is Vehicle A Less Detectable Than Vehicle B?	
Decision:	No	Yes
Accept Null Hypothesis	Correct	Type II Error
Accept Alternative Hypothesis	Type I Error	Correct

Table 1. Definitions of Type I and Type II errors.

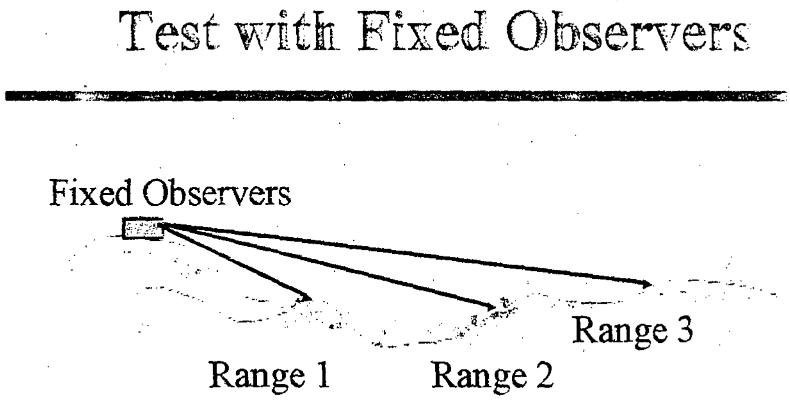

Figure 1. Experimental setup for an observer test.

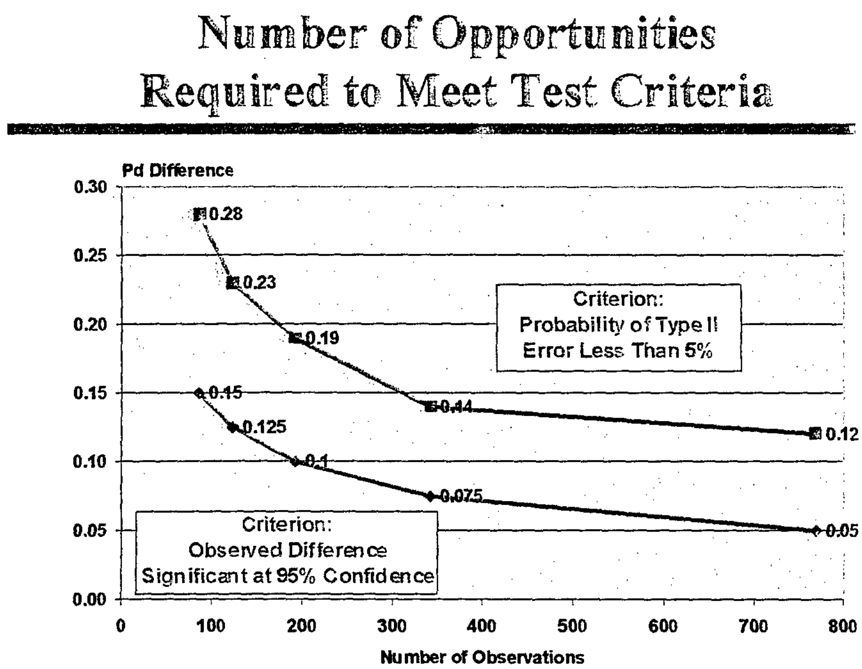

Figure 2. Number of observation opportunities required to meet test criteria.

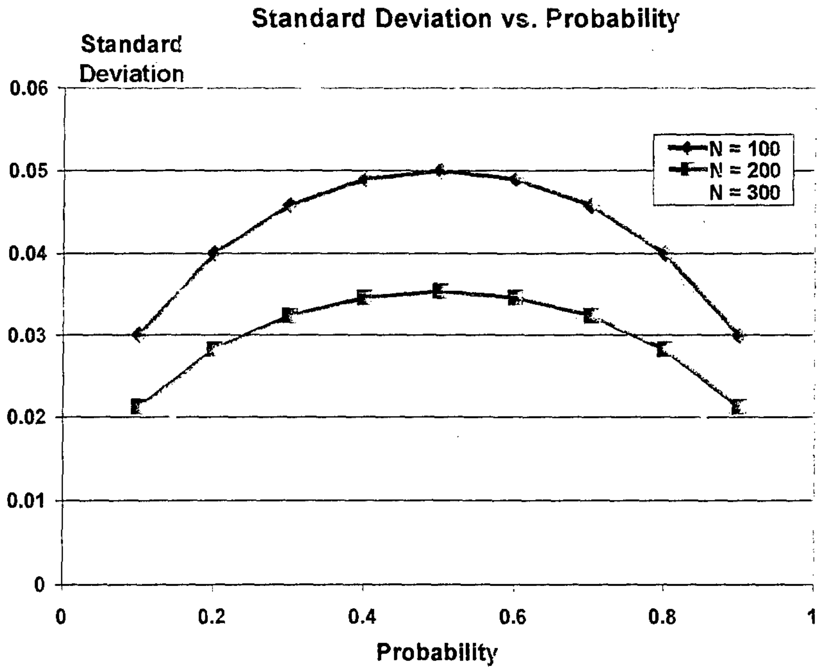

Figure 3. Standard deviation of a binary distribution for 100, 200 and 300 trials.

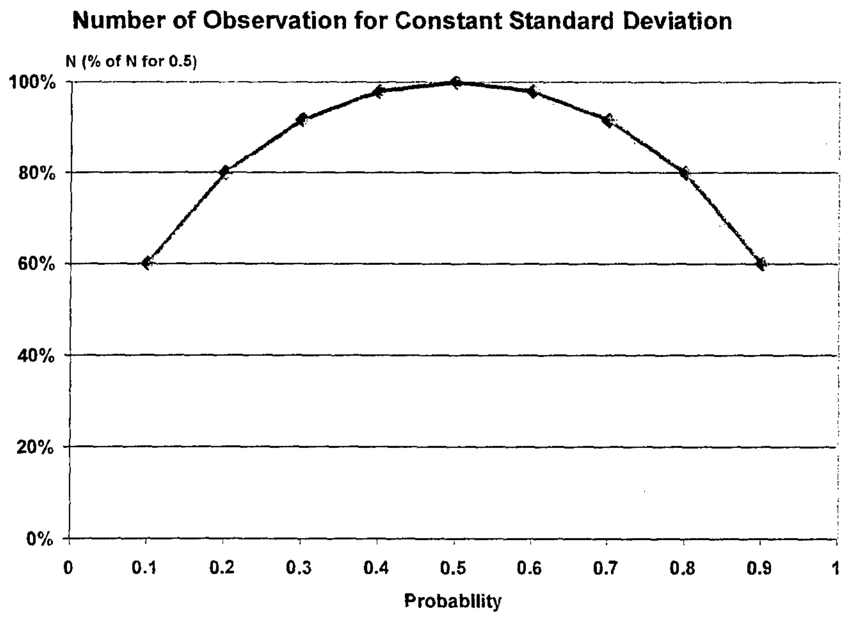

Figure 4. Standard deviation of a binary distribution normalized to probability of 0.5.

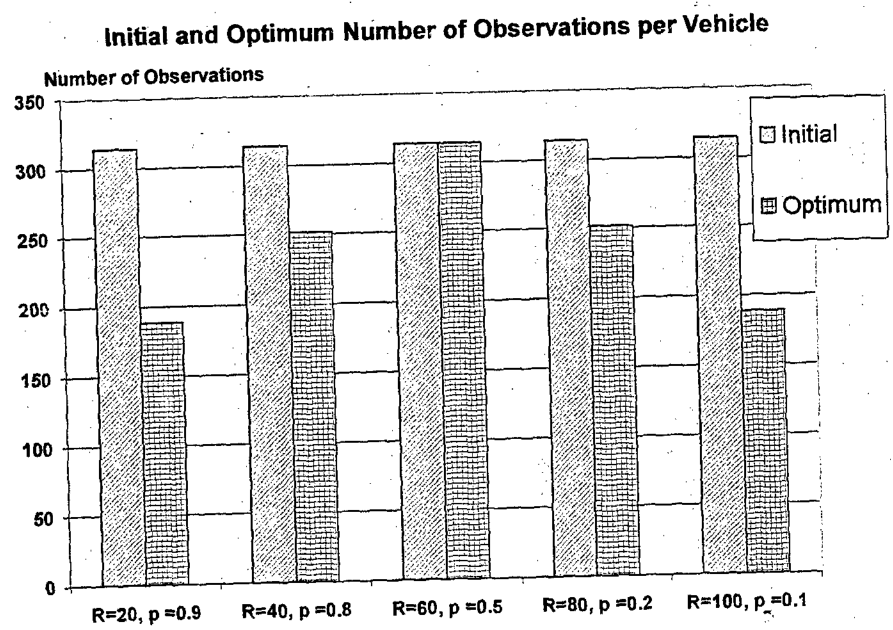

Figure 5. An example of iterative design of an observer test.